ITEP-TH-28/99

Classical String Solutions in Effective Infrared Theory of SU(3) Gluodynamics

M.N. Chernodub

Institute of Theoretical and Experimental Physics, B.Cheremushkinskaya 25, Moscow, 117259, Russia

Abstract

We investigate string solutions to the classical equations of motion ("classical QCD strings") for a dual Ginzburg-Landau model corresponding to $SU(3)$ gluodynamics in an abelian projection. For a certain relation between couplings of the model the string solutions are defined by first order differential equations. These solutions are related to vortex configurations of the Abelian Higgs model in the Bogomol'ny limit. An analytic expression for the string tension is derived and the string–string interactions are discussed. Our results imply that the vacuum of $SU(3)$ gluodynamics is near a border between type-I and type-II dual superconductivity.

1. Last years the problem of color confinement in $SU(N)$ Yang–Mills theories has been intensively studied in the framework of the abelian projection approach [1]. This approach is based on a partial gauge fixing which reduces the gauge symmetry from non–abelian gauge group to its (maximal) abelian subgroup. The diagonal elements of the gluon field transform under residual gauge transformations as abelian gauge fields, while the off-diagonal elements transform as abelian matter vector fields. Due to compactness of the Yang–Mills gauge group the residual abelian subgroup is also compact. This leads to appearance of abelian monopoles in the abelian gauge. According to the dual superconductor scenario [2] the color confinement may be explained at the classical level: if monopoles are condensed then a string forms between color charges. This string is an analog of the Abrikosov string [3] in a superconductor and the abelian monopoles are playing the role of the Cooper pairs.

The dual superconductor mechanism has been confirmed by numerous lattice simulations of $SU(2)$ Yang–Mills theory [4]. Moreover, the vacuum of $SU(2)$ Yang–Mills theory in the abelian projection was shown [5] to be close to the border between type–I and type–II dual superconductors (masses of the monopole and the dual gauge boson are approximately the same). There are also strong numerical indications that the vacuum of $SU(3)$ gluodynamics exhibits dual superconductor properties [6]. Analytical investigations [7] of the string configurations in a non–abelian dual superconductor model of $SU(3)$ gluodynamics shows that the dual vacuum is close to the border between type-I and type-II superconductivity. In Ref. [8] similar conclusion has been derived where the $SU(3)$ confining string was related to the Abrikosov–Nielsen–Olesen classical string solution [3, 9] in the $U(1)$ Higgs model.

In this paper we investigate an effective *abelian* model of $SU(3)$ gluodynamics suggested in Ref. [10]. This model is based on dual superconductivity conjecture which is supposed to describe infrared properties of the $SU(3)$ gluodynamics vacuum. An essential difference between $SU(2)$ and $SU(3)$ Yang–Mills theories in an abelian projection is the presence of two independent string configurations in the later. The existence of two string types guarantees the colour neutrality of the quark bound states [11]. A possible string representation of this model in a regime of extreme type-II superconductivity (the monopole mass is infinite) has been discussed in Refs. [11, 12]. The classical string configurations in the static baryon have been studied numerically in Refs. [13, 14]. The results of Ref. [14] suggest that the vacuum of $SU(3)$ gluodynamics lies near a border between type–I and type–2 superconductivity. Below we study analytically the confining string regarding the $SU(3)$ gluodynamics as a dual superconductor with finite masses of the monopole and dual gauge boson fields. A special attention is payed to the border between different types of superconductivity.

2. The Lagrangian of $[U(1)]^2$ Higgs model corresponding to $SU(3)$ gluodynamics is[1] [10]:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_{i=1}^{3} \Bigl[\frac{1}{2} {|D^{(i)}_\mu \chi_i|}^2 - \lambda \Bigl({|\chi_i|}^2 - \eta_i^2 \Bigr)^2 \Bigr] \,, \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu$ is the field strength for the gauge fields B^a_μ, $a = 3, 8$, $D^{(i)}_\mu = \partial_\mu + i g \varepsilon^a_i B^a_\mu$ is the covariant derivative acting on the Higgs fields χ_i, $i = 1, 2, 3$. The ϵ's are the root vectors of the group $SU(3)$: $\vec{\epsilon}_1 = (1, 0)\,, \vec{\epsilon}_2 = (-1/2, -\sqrt{3}/2)\,, \vec{\epsilon}_3 = (-1/2, \sqrt{3}/2)$.

The gauge fields $B^{3,8}_\mu$ are dual to the diagonal components $a = 3, 8$ of the gluon field A^a_μ. The Higgs fields χ_i are associated with the monopole degrees of freedom which appear due to compactness of the residual abelian gauge group, $[U(1)]^2$, in an abelian projection $SU(3) \to [U(1)]^2$. Lagrangian (1) respects the dual $[U(1)]^2$ gauge invariance: $B^a_\mu \to B^a_\mu + \partial_\mu \alpha^a$, $\theta_i \to \theta_i + g(\varepsilon^3_i \alpha^3 + \varepsilon^8_i \alpha^8) \bmod 2\pi$, $a = 3, 8$, $i = 1, 2, 3$, where α^3 and α^8 are the parameters of the gauge transformation. The phases of the Higgs fields satisfy the following relation:

$$\sum_{i=1}^{3} \arg \chi_i = 0 \,. \tag{2}$$

The model admits vortex–like solutions to the classical equations of motion similar to the Abrikosov vortex configurations in the Ginzburg–Landau model of superconductivity. The vortices carry quantized fluxes of electric fields since the matter fields χ_i correspond to the magnetically charged particles. We choose the following anzatz for the field configuration of the static straight vortex parallel to the z–axis:

$$\begin{array}{rcll} \chi_i & = & \eta_i\, f_i(\rho) e^{- i n_i \varphi}\,, & n_i \in Z\!\!\!Z\,, \quad i = 1, 2, 3\,, \\ \vec{B}^a & = & -\frac{1}{g\rho} v^a(\rho) \cdot \vec{e}_\varphi\,, & a = 3, 8\,. \end{array} \tag{3}$$

where we have used the standard notations for the cylindrical coordinates (φ, ρ, z, t) in four dimensional space. Note that the vortex configuration does not depend on z– and t–coordinates.

[1] In this paper the summation over the Latin indices i and j is taken only if explicitly indicated.

The finite energy configuration must satisfy the condition $D^{(i)}_\mu \chi_i = 0$ at spatial infinity. This condition implies the quantization of the total flux $\Phi^a = \int \mathrm{d}x\mathrm{d}y\, H^a$ of the vortex magnetic field $H^a = F^a_{12}$:

$$\Phi_i \equiv \sum_{a=3,8} \varepsilon^a_i\, \Phi^a = \frac{2\pi n_i}{g} \quad \text{or} \quad \Phi^a = \frac{4\pi}{3g} \sum_{i=1}^{3} \varepsilon^a_i n_i\,, \tag{4}$$

where we have used the relation $\sum_{i=1}^3 \epsilon^a_i \epsilon^b_i = 3\delta^{ab}/2$. In particular this condition implies that the strings must carry both components, Φ^3 and Φ^8, of the fluxes.

According to eq.(2) the integer winding numbers n_i satisfy the following relation:

$$\sum_{i=1}^{3} n_i = 0\,. \tag{5}$$

3. To analyse the vortex solutions we begin with the Bogomol'ny method [15]. Consider the energy density per unit length of the string (string tension):

$$\sigma = \int \mathrm{d}^2 x \,\Big[\frac{1}{2}(H^a)^2 + \sum_{i=1}^{3}\Big(\frac{1}{2}\sum_{\alpha=1,2} \big|D^{(i)}_\alpha \chi_i\big|^2 + \lambda\Big(|\chi_i|^2 - \eta_i^2\Big)^2\Big)\Big]\,. \tag{6}$$

The first term in this equation can be rewritten as follows: $\sum_{a=1}^2 (H^a)^2/2 = \sum_{i=1}^3 {H_i}^2/3$, where $H_i = \sum_{a=1}^2 \epsilon^a_i H^a$. The second term in eq.(6) can be represented as follows:

$$\sum_{\alpha=1,2} \big|D^{(i)}_\alpha \chi_i\big|^2 = \Big|\Big(D^{(i)}_1 \pm i D^{(i)}_2\Big)\chi_i\Big|^2 \mp 2\varepsilon_{\alpha\beta}\partial_\alpha J^{(i)}_\beta \pm i\chi^+_i \Big[D^{(i)}_1, D^{(i)}_2\Big]\chi_i\,, \tag{7}$$

where the current $J^{(i)}_\alpha = i\chi^+_i D^{(i)}_\alpha \chi_i$ vanishes faster than ρ^{-1} as $\rho \to \infty$ for the configuration with a finite energy. Therefore the second term in the *r.h.s.* of eq.(7) gives zero contribution to the string tension (6). The last term in eq.(7) can be simplified: $i\chi^+_i [D^{(i)}_1, D^{(i)}_2]\chi_i = -gH_i\,|\chi_i|^2$. Thus the string tension (6) can be expressed in the form:

$$\begin{aligned}
\sigma &= \int \mathrm{d}^2 x \sum_{i=1}^{3}\Big[\frac{1}{2}\Big|\Big(D^{(i)}_1 \pm i D^{(i)}_2\Big)\chi_i\Big|^2 + \frac{1}{3}H_i^2 \mp \frac{g}{2} H_i\,|\chi_i|^2 + \lambda\Big(|\chi_i|^2 - \eta_i^2\Big)^2\Big] \\
&= \int \mathrm{d}^2 x \sum_{i=1}^{3}\Big[\frac{1}{2}\Big|\Big(D^{(i)}_1 \pm i D^{(i)}_2\Big)\chi_i\Big|^2 + \frac{1}{3}\Big(H_i \mp \frac{3g}{4}(|\chi_i|^2 - \eta_i^2)\Big)^2 \\
&\quad \pm \frac{g}{2}\eta_i^2 \Phi_i + \Big(\lambda - \frac{3g^2}{16}\Big)\Big(|\chi_i|^2 - \eta_i^2\Big)^2\Big]\,,
\end{aligned} \tag{8}$$

where the magnetic flux Φ_i is quantized according to eq.(4). The sign in front of the flux term is chosen so that the contribution of this term is positive.

The analogue of the Bogomol'ny limit [15] is defined by the following condition:

$$\lambda = 3g^2/16\,, \tag{9}$$

which guarantees that the potential term in eq.(8) vanishes. Contrary to the case of Abelian Higgs model this condition does not imply an equivalence of the vector and scalar boson masses. The equivalence holds only in the physically relevant case when the vacuum expectation values of the Higgs monopole fields χ_i are degenerate, $\eta_i \equiv \eta_j$, $i,j = 1,2,3$ (the color symmetry is unbroken).

In this case the vortex configuration is defined by the first order field equations:

$$\Bigl(D^{(i)}_1 \pm i D^{(i)}_2\Bigr)\chi_i = 0\,; \quad H_i \mp \frac{3g}{4}\Bigl(|\chi_i|^2 - \eta^2_i\Bigr) = 0\,; \quad i = 1,2,3\,, \tag{10}$$

or, for anzatz (3),

$$f'_i(\rho) \pm (v_i(\rho) - n_i) f_i(\rho) \rho^{-1} = 0\,, \quad \pm v'_i(\rho) + g^2 \eta^2_i (f^2_i(\rho) - 1)\rho = 0\,, \tag{11}$$

where $v_i = \sum_{a=3,8} \varepsilon^a_i v^a$ and the prime denotes the differentiation with respect to ρ. The system of equations of motion (11) is over-defined and therefore a classical vortex solution to these equations does not exist in general case. However we will show below that for a physical case the classical solution can be found.

According to eqs.(4,8) the solutions to the equations of motion (10) or (11) saturate an analog of the Bogomol'ny bound for the string tension:

$$\sigma = \pi \sum^3_{i=1} |n_i| \eta^2_i\,. \tag{12}$$

Thus parallel strings which carry the same flux do not interact with each other since energy of N strings with the same flux Φ^a is the same as the energy of the single string with the flux $N\Phi^a$. This is a general property of classical solutions in the Bogomol'ny limit. The vortices start to interact when relation (9) is not satisfied. A similar situation happens on a border between type-I and type-II superconductors.

4. The $SU(3)$ string in an abelian projection may be considered as a composite [11, 12] of the three *elementary* strings with the winding numbers n_i of the Higgs fields χ_i subjected to relation (5). The phase of the Higgs field must be singular at the center of the elementary string with non-zero winding number n_i. Since all the Higgs field phases are subjected to the condition (2) the center of the elementary strings must coincide. The tension of the $SU(3)$ string depends on the fluxes Φ^a carried by the string and the fluxes are being related to the winding numbers n_i in accordance with the quantization condition (4).

Consider the classical field configuration for the abelian counterpart of the $SU(3)$ string in the Bogomol'ny limit. The lowest energy string configuration in the neutral quark–anti-quark system has the winding numbers 1, -1 and 0 (there are three string configurations corresponding to different colors of the quarks, or, equivalently, to different permutations of the n's). Note that the same winding numbers are carried by segments of the $SU(3)$ classical string configuration in the baryon [11].

For the sake of generality we consider below the classical string solution with the winding numbers n, $-n$ and 0. The solution to the equations of motion (11) is:

$$f_i(\rho) = f^{\mathrm{ANO}}_{|n_i|}(\rho, g\eta_i)\,, \quad v^a = \frac{2}{3} \sum^3_{i=1} \mathrm{sign}(n_i)\, \varepsilon^a_i\, v^{\mathrm{ANO}}_{|n_i|}(\rho, g\eta_i)\,, \tag{13}$$

where $f_n^{\rm ANO}(\rho,m)$ and $v_n^{\rm ANO}(\rho,m)$ are the characteristic functions of the Abrikosov–Nielsen–Olesen (ANO) vortex solution [3, 9] in the Bogomol'ny limit of the Abelian Higgs model (AHM). These functions are defined by the first order Bogomol'ny equations in the AHM [16]:

$$f_n^{\rm ANO\prime} \pm (v_n^{\rm ANO} - n) f_n^{\rm ANO} \rho^{-1} = 0\,, \quad \pm v_n^{\rm ANO\prime}(\rho) + m^2 \Bigl((f_n^{\rm ANO})^2 - 1 \Bigr) \rho = 0\,, \quad n > 0\,. \tag{14}$$

The functions $f_n^{\rm ANO}$ and $v_n^{\rm ANO}$ have been determined numerically in Ref. [16]. Note that $f_0^{\rm ANO} = 1$ and $v_0^{\rm ANO} = 0$.

In the degenerate case $\eta_i = \eta$, $i = 1,2,3$, the characteristic functions v^a for the string with the winding numbers n, $-n$ and 0 have a simple form:

$$v^a(\rho) = \frac{\Phi^a}{2\pi n}\, v_n^{\rm ANO}(\rho, g\eta)\,, \quad n > 0\,, \tag{15}$$

where the fluxes Φ^a are given by eq.(4) and according to eq.(12) the string tension is

$$\sigma_n = 2\pi n \eta^2\,. \tag{16}$$

The solutions for a general case ($n_i \neq 0$, $i = 1,2,3$) are unlikely to exist due the fact that system (11) is over-defined. Indeed according to the definition of the vectors v_i we need the fulfillment of the following condition: $\sum_{i=1}^3 n_i\, v_{|n_i|}^{\rm ANO}(\rho, g\eta_i) = 0$ with given vorticities n_i and *integer* parameters η_i. Since the functions $v_n^{\rm ANO}$ are solutions of the nonlinear equations the linear dependence of these functions would be unnatural. Thus the Bogomol'ny bound (12) for the string tension is likely to be reached only for the strings composed of two elementary abelian strings (the third vorticity component must be zero).

5. The interaction between vortices is defined by their Higgs and gauge boson field profiles. The Higgs (gauge) boson mediated interaction between two parallel identical vortices is attractive (repulsive). In the Bogomol'ny limit the Higgs and gauge boson fields acts in the same range since at large distances the profile functions $v^{\rm ANO}(\rho, m_1)$ and $f^{\rm ANO}(\rho, m_2)$ behave like $e^{-m_1\rho}$ and $e^{-m_2\rho}$, respectively. According to eq.(13) the mass m_1 equals to the mass m_2 in the Bogomol'ny limit. Thus the interactions compensate each other and the net force between vortices is zero[2].

When $\lambda < 3g^2/16$ the vortices are attracting since the attraction due to exchange of the Higgs boson prevails over the gauge boson attraction. In this region of the parameter space the strings with identical fluxes tend to join. For $\lambda > 3g^2/16$ the strings repel each other and therefore the strings with multiple vorticities are unstable: they tend to decay on strings with smallest vorticities.

Using similar arguments one can show that the strings with different fluxes (winding numbers) are always attractive in the type-I region. In the type-II region the gauge–mediated force between strings A and B is proportional to $\sum_{a=1,2} \Phi^a_A \Phi^a_B \propto \sum_{i=1}^3 n_i^A n_i^B$. The interaction between the stings is attractive if this number is negative and repulsive otherwise.

[2]Note that the $[U(1)]^2$ vortices are not interacting for all vortex–vortex separations contrary to, *e.g.*, $\mathbb{Z}_3$ vortices (these are also supposed [17] to be relevant to confinement in QCD.). The potential between $\mathbb{Z}_3$ vortices may be attractive at short separations and repulsive at large distances [18].

In an ordinary type–I (type–II) superconductor the Abrikosov vortices attract (repel) each other and the Bogomol'ny limit is a border line between the two types. By analogy, relation (9) defines the border line between different types of the dual superconductivities.

The closeness of the $SU(3)$ gluodynamics vacuum to the border between type-I and type-II may be checked with the help of eq.(16) which relates vacuum expectation values of the monopole fields η and the string tension *at the border*. Using the estimate [10, 13, 14, 19] for the vacuum expectation value of the Higgs fields, $\eta \approx 175$ MeV, we get[3] the value of the tension of the string with the lowest flux, $\sigma \equiv \sigma_1 \approx (440 \text{ MeV})^2$ which is quite close to a phenomenological value. Thus the gluodynamics vacuum is likely to be near the border between type-I and type-II dual superconductivity.

Conclusions

The effective model of $SU(3)$ gluodynamics in an abelian projection possesses classical vortex solutions ("QCD strings"). For a certain relation between couplings of this model ($16\lambda = 3g^2$) the vortex solutions are related to the solutions of the Abelian Higgs model in the Bogomol'ny limit. This relation also defines the border between type-I and type-II dual superconductivity. The string tension in the Bogomol'ny limit is quantized according to eq.(12). Using this formula and the phenomenological value of the vacuum expectation value of the monopole field we show that the vacuum of the $SU(3)$ gluodynamics lies near the border between type–I and type–II dual superconductivity.

Acknowledgments

The author is grateful to M.I. Polikarpov, T. Suzuki and T. Vachaspati for useful discussions. The author feel much obliged for the kind hospitality extended to him at the Theoretical Physics Institute at the University of Minnesota where this work has been started. This work was supported by the grants INTAS-96-370, RFBR-96-15-96740, RFBR-99-01-01230 and INTAS Grant 96-0457 within the research program of the International Center for Fundamental Physics in Moscow.

References

[1] G. 't Hooft, *Nucl. Phys.* **B190[FS3]** (1981) 455.

[2] Y. Nambu, *Phys. Rev. D10* (1974) 4262;
A.M. Polyakov, *Phys. Lett.* **59B** (1975) 82;
S. Mandelstam, *Phys. Rep.* **23C** (1976) 245;
G. 't Hooft, in *High Energy Physics, Proceedings of the EPS International Conference*, Palermo, Italy, 23-28 July 1975, ed. by A. Zichichi, Editrice Compositori, Bologna, 1976.

[3] Note that the notations of Refs. [10, 13, 14, 19] differ from our notations.

[3] A.A. Abrikosov, *Sov. Phys. JETP* **5** (1974) 1174.

[4] M.I. Polikarpov, *Nucl. Phys. Proc. Suppl.* **53** (1997) 134;
M.N. Chernodub and M.I. Polikarpov, in *"Confinement, Duality and Non-perturbative Aspects of QCD"*, p.387, Ed. by Pierre van Baal, Plenum Press, 1998; `hep-th/9710205`;
G.S. Bali, *preprint HUB-EP-98-57, Jan. 1998*, `hep-ph/9809351`.

[5] Y. Matsubara, S. Ejiri and T. Suzuki, *Nucl. Phys. Proc. Suppl.* **34** (1994) 176;
V. Singh, D.A. Browne and R.W. Haymaker, *Phys. Lett.* **B306** (1993) 115;
G. S. Bali, C. Schlichter and K. Schilling, *Prog. Theor. Phys. Suppl.* **131** (1998) 645.

[6] F. Brandstater, U.J. Wiese and G. Schierholz, *Phys. Lett.* **B272** (1991) 319;
Y. Matsubara, S. Ejiri and T. Suzuki, *Nucl. Phys. Proc. Suppl.* **34** (1994) 176;
Ken Yee, *Nucl. Phys. Proc. Suppl.* **34** (1994) 189;
N. Arasaki *et al.*, *Phys. Lett.* **B395** (1997) 275.

[7] M. Baker, J.S. Ball and F. Zachariasen, *Phys. Rev.* **D31** (1985) 2575;
M. Baker, James S. Ball and F. Zachariasen, *Phys. Rev.* **D41** (1990) 2612.

[8] M. Baker *et al.*, *Phys. Rev.* **D58** (1998) 034010.

[9] H.B. Nielsen and P. Olesen, *Nucl. Phys.* **B61** (1973) 45.

[10] S. Maedan and T. Suzuki, *Prog. Theor. Phys.* **81** (1989) 229.

[11] M. N. Chernodub and D. A. Komarov, *JETP Lett.* **68** (1998) 117.

[12] D. Antonov and D. Ebert, *Phys.Lett.* **B444** (1998) 208.

[13] S. Kamizawa *et al.*, *Nucl. Phys.* **B389** (1993) 563.

[14] S. Maedan, Y. Matsubara and T. Suzuki, *Prog. Theor. Phys.* **84** (1990) 130.

[15] E. B. Bogomol'ny, *Sov. J. Nucl. Phys.* **24** (1976) 449.

[16] H.J. de Vega and F.A. Schaposnik, *Phys. Rev.* **D14** (1976) 1100.

[17] T. Vachaspati, *Phys. Rev. Lett.* **76** (1996) 188.

[18] J. Heo and T. Vachaspati, *Phys. Rev. D58* (1998) 065011.

[19] H. Suganuma, S. Sasaki and H. Toki, *Nucl. Phys.* **B435** (1995) 207.